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Stats 2 – June 2011

(1) a) i) $2.6 \times 5 = 13 \rightarrow X \sim Po(13)$

ii) $P(X = 20) = \dfrac{e^{-13} \times 13^{20}}{20!} = 0.0177$

iii) $P(6 \leq X \leq 18) = P(X \leq 18) - P(X \leq 5)$

$= 0.9302 - 0.0107 = 0.9195$

b) Number of cars passing by is not random
Cars are not independent of each other
Mean likely to change at rush hour

c) $Y \sim B(20, 0.2)$

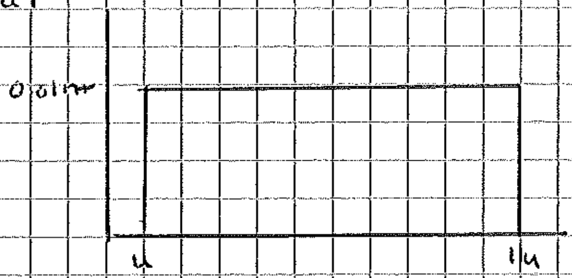
$P(Y \geq 5) = 1 - P(Y \leq 4)$ (from tables)

$= 1 - 0.6296 = 0.3704$

d) Assumption: X & Y are independent

prob $= 0.0177 \times 0.3704 = 0.00655608$

(2) a)



i) Area $= 1$

$\rightarrow 10u \times 0.01\pi = 1$

$\rightarrow \frac{1}{10} u\pi = 1$

$\rightarrow u = \frac{10}{\pi}$

ii) $E(X) = \frac{1}{2}(11u + u) = 6u = 6\left(\frac{10}{\pi}\right) = \frac{60}{\pi}$

$Var(X) = \frac{1}{12}(11u - u)^2 = \frac{1}{12}(100u^2)$

$= \frac{100}{12}\left(\frac{10^2}{\pi^2}\right) = \frac{10{,}000}{12\pi^2}$

iii) $C = \pi \times d$

$\rightarrow C = \pi\left(X + \frac{10}{\pi}\right) = \pi X + 10$

$\therefore E(C) = \pi E(X) + 10$

$= \pi\left(\frac{60}{\pi}\right) + 10 = 70$

$Var(C) = \pi^2 Var(X)$

$= \pi^2\left(\frac{10{,}000}{12\pi^2}\right) = \frac{10{,}000}{12} = 833\frac{1}{3}$

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b) $n = 100$

$\bar{y} = 40.5$

$\sigma^2 = 25$

$n > 100$, so use z

95%, 2 tailed $\Rightarrow$ $z = 1.96$

$\therefore$ 95% CI $= 40.5 \pm 1.96 \times \sqrt{\frac{25}{100}}$

$= 40.5 \pm 0.98$

$= (39.52, 41.48)$

(3) a) H_0: no association between type of school and performance at GCSE

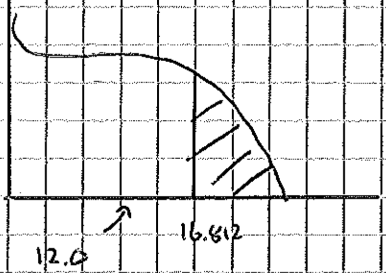
b) Test stat: Need $\frac{(O-E)^2}{E}$

$\rightarrow$ 0.19581...	0.0625...	0.54185...
0.48216...	1.2694...	0.06401...
0.00356...	0.78549...	3.27410...
1.0805...	0.18380...	4.09642...

$\sum X^2 = 12.0197... = 12.0$ (1dp)

c) $v = (4-1) \times (3-1) = 6$

Critical value: $X^2_{(6)}$ 1% $= 16.812$



$12.0 < 16.812$

$\therefore$ Accept H_0

No significant evidence to support Emily's belief of an association

d) Fewer than expected gained no GCSEs

e) $X^2_{(6)}$ 10% critical value $= 10.645$

$10.645 < 12.0$

$\therefore$ Reject H_0

Evidence to support Emily's belief of an association

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(4) a)

x	1	2	3	4	5
$P(X=x)$	$3/40$	$6/40$	$9/40$	$12/40$	$5/20$

$E(X) = 1 \times 3/40 + 2 \times 6/40 + 3 \times 9/40 + 4 \times 12/40 + 5 \times 5/20$

$= 3.5$

b) i) $E(1/x) = 1 \times 3/40 + 1/2 \times 6/40 + 1/3 \times 9/40 + 1/4 \times 12/40 + 1/5 \times 5/20$

$= 7/20$

ii) $E(1/x^2) = 1^2 \times 3/40 + (1/2)^2 \times 6/40 + (1/3)^2 \times 9/40$

$+ (1/4)^2 \times 12/40 + (1/5)^2 \times 5/20 = 133/800$

$\therefore Var(1/x) = 133/800 - (7/20)^2 = 7/160$

c)

x	1	2	3	4	5
y	40	20	$40/3$	10	8
prob	$3/40$	$6/40$	$9/40$	$12/40$	$5/20$

i) $P(Y < 20) = P(x = 3, 4, 5) = 9/40 + 12/40 + 5/20 = 31/40$

ii) $P(X < 4 / Y < 20) = \frac{P(X < 4 \text{ AND } Y < 20)}{P(Y < 20)}$

Only happens when $x = 3$ → $= \frac{9/40}{31/40} = 9/31$

(5) a) $H_0: \mu = 20{,}000$

$H_1: \mu \neq 20{,}000$ (2 tailed)

$n = 25$

$\bar{x} = 19{,}700$

$\sigma = 640$

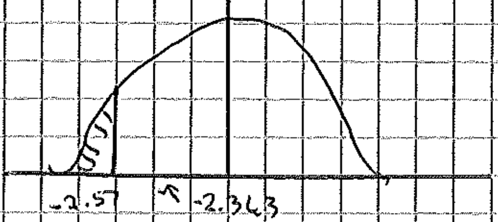
we know σ so can use Z

Test Statistic: $z = \frac{19700 - 20{,}000}{640/\sqrt{25}} = -2.34375$

Critical Value: 1%, 2 tailed → $z = \pm 2.5758$

$-2.343 > -2.575$

$\therefore$ Accept H_0



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Not enough evidence to suggest mean life of lightbulb has changed.

b) i) $H_1: \mu < 10{,}000$ (1 tailed)

ii) $n = 16$

don't know σ, so must use t

$s = 500$

$v = 16 - 1 = 15$

critical value: t, 5%, $v = 15$, 1 tailed

$= 1.753$

would <u>not</u> reject if: $\bar{x} \pm 1.753 \times \frac{500}{\sqrt{16}}$

$\rightarrow 10{,}000 \pm 1.753 \times \frac{500}{\sqrt{16}}$

$= 10{,}000 \pm 219.125$

$= (9780.875,\ 10{,}219.125)$

would <u>not</u> reject if $\bar{x}$ was > 9780 (as 1 tailed test)

iii) No error as H_0: is $\mu = 10{,}000$

⑥ a) $F(x) = \int f(x)\, dx$

$= \int_0^x \frac{3}{8}(x^2 + 1)\, dx = \frac{3}{8} \int_0^x x^2 + 1\, dx$

$= \frac{3}{8}\left[\frac{x^3}{3} + x\right]_0^x$

$= \frac{3}{8}\left[\left(\frac{x^3}{3} + x\right) - 0\right] = \frac{1}{8}x^3 + \frac{3}{8}x$

$= \frac{1}{8}x(x^2 + 3)$

b) $F(m) = \frac{1}{2}$

$F(1) = \frac{1}{8}(1)(1^2 + 3) = \frac{1}{8}(4) = \frac{1}{2}$ $\therefore 1 =$ median

c) UQ must lie in $1 \leq x \leq 2$

Need $F(UQ) = 0.75$

$\int_1^q \frac{1}{4}(5 - 2x)\, dx = \frac{1}{4}\int_1^q 5 - 2x\, dx$

$= \frac{1}{4}\left[5x - x^2\right]_1^q = \frac{1}{4}\left[(5q - q^2) - (5 - 1)\right]$

$= \frac{1}{4}\left[5q - q^2 - 4\right]$

must add on $F(1) = \frac{1}{2}$

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$\Rightarrow \frac{1}{2} + \frac{1}{4}[5q - q^2 - 4] = 0.75$

$\Rightarrow \frac{1}{4}[-q^2 + 5q - 4] = 0.25$

$\Rightarrow -q^2 + 5q - 4 = 1$

$\Rightarrow q^2 - 5q + 5 = 0$

Solve using formula:

$\Rightarrow q = \dfrac{5 \pm \sqrt{5^2 - 4 \times 1 \times 5}}{2}$

$= \dfrac{5 \pm \sqrt{5}}{2}$ but q must be between 1 & 2

$\Rightarrow q = \dfrac{5 - \sqrt{5}}{2} = \frac{1}{2}(5 - \sqrt{5})$

d) $P(q < X < 1.5)$

$= P(\frac{1}{2}(5 - \sqrt{5}) < X < 1.5)$

~~① P(X < 1.5)~~

For $P(X < 1.5)$ use $F(x) = \frac{1}{2} + \frac{1}{4}[-x^2 + 5x - 4]$

$\Rightarrow F(1.5) = \frac{1}{2} + \frac{1}{4}(-1.5^2 + 5(1.5) - 4]$

$= 13/16$

For $P(X < q) = 3/4$ (as it's $u(q)$)

$\therefore P(q < X < 1.5) = 13/16 - 3/4 = 1/16$